Journal of Experimental Psychology:
Human Perception and Performance
1979, Vol. 5, No. 2, 289-302

Perception of Wheel-Generated Motions

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Data are presented on an old and familiar Gestalt demonstration—perceiving wheel-generated motions—in which the perceived motions of a rolling wheel are shown not to be obviously derived from the motions of the parts. The history of study of this phenomenon is presented, and contradictions in the literature are noted. The focus for experimentation is on the contrasting approaches found in Johansson's perceptual vector analysis and Wallach's arguments for the priority of object-relative displacement in the extraction of invariants. Johansson's approach asserts that common vectors are extracted from moving events first, whereas Wallach asserts that the motion of objects relative to each other is first. These two approaches yield different predictions about what ought to be seen when different configurations are viewed in rotation. In five experiments viewers rated how wheellike the movement of various point-light systems attached to a rolling wheel appeared to be. Results support Wallach's views over Johansson's. Viewer judgments of goodness in wheellike motion correspond highly with a mathematical description of the parameters of cycloidal motion for the geometric center of any system of lights on a rolling wheel. This specification can be made only after the extraction of object-relative displacement information. Number of lights and order of symmetry influence viewer judgments to a much lesser degree, and placement of a light at the wheel's center matters not at all.

Several Gestalt phenomena have become classic examples of the extraction of higher order invariants by the perceptual system. The perception of wheel-generated motions is one of the most familiar. This particular phenomenon has drawn attention because the perceived motions of a rolling wheel are not obviously manifested in the motions of the wheel's individual parts. Except for the linear motion of the center, all points on a rolling wheel move along paths describing cycloids or prolate cycloids. Rather than perceiving these curves, we see points on a wheel revolving about the center and the whole moving with a linear motion. In the top panel of Figure 1 is a diagram of the motion of a rolling wheel with the center represented as a small dot and a perimeter light as a large dot. With surround darkened, the path of the moving light is seen to describe a cycloid. Our typical perception of rolling wheels is shown in the lower panel of Figure 1, which represents any perimeter point revolving about the center that moves linearly.

Psychologists have been fascinated with this phenomenon for over a half century; however, some of their accounts possess incompatibilities. For this reason, we initiated a series of five experiments, and in particular, focused our investigation by constrasting Johansson's (1973) perceptual vector

This research was supported by grants to authors Proffitt and Cutting from the Wesleyan University. We thank C. Stewart Gillmor of the Wesleyan University Department of History and Howard Bernstein of the Yale University Department of History of Science and Medicine for their assistance on the mathematical history of the cycloid, and Ethan M. Coven of the Wesleyan University Department of Mathematics for his help with symmetry theory. Robert J. White provided invaluable technical assistance.

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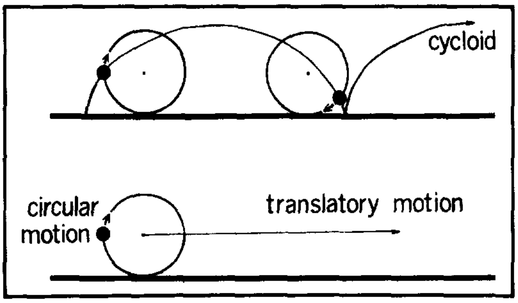

Figure 1. The upper panel shows the trace of a light mounted on the perimeter of a rolling wheel. The lower panel shows the circular and translatory components of motion. One light mounted on a wheel is insufficient to perceive wheellike motion.

analysis with Wallach's (1965/1976) discussion of the phenomenon.

The nonobvious nature of cycloidal curves is evidenced by the fact that they were not discovered by mathematicians until the seventeenth century. A brief sketch is informative, demonstrating the enormous significance that the cycloid was found to have by the mathematicians of this period.

The Helen of the Geometers

It does not appear possible to determine who first discovered the cycloid. The Greeks seem to have been completely unaware of it. Boyer (1968, p. 173) writes,

> Aesthetically one of the most gifted people of all times, the only curves that they [the Greeks] found in the heavens and on earth were combinations of circles and straight lines . . . Even the cycloid, generated by a point on a circle that rolls along a straight line, seems to have escaped their notice. That Apollonius, the greatest geometer of antiquity, failed to develop analytic geometry, was probably the result of a poverty of curves rather than of thought.

Although there is some controversy about who should get credit for discovering the cycloid, there is agreement that Galileo knew of it, and he is credited with giving it its name. Rubin (1927) reports being told that Galileo noticed the cycloid at a peasant festival. The peasants were rolling wagon wheels down a hill at night, and attached to each wheel was a torch.[1] About 1630 Mersenne brought the cycloid to the attention of French mathematicians and suggested its importance. "It soon became one of the most discussed curves of the period, the discussions occasionally leading to acrimonious remarks, so that the curve has been compared to the apple of discord or called the Helen of the geometers" (Struik, 1969, p. 232). At this time the cycloid was frequently called a *roulette,* or *trochoid* after the Greek *trochos,* wheel. Roberval effected the quadrature of the cycloid in 1634, but mathematical descriptions of the curve culminated with Leibniz, who wrote its equation in 1686. (See Whitman, 1943, for an account of the history of the mathematical study of the cycloid.)[2]

Perceiving Rotary Motions

Among psychologists, Rubin (1927) was first to propose that one does not see the

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[1] This is such a delightful story that it ought to be true; however, we have thus far been unable to confirm it. Galileo tried to determine the area under one arch of the cycloid, its quadrature, but failed because the required mathematics had not been formulated. Besides the Galileo story, Rubin (1927) noted that the philosopher Sigwart (1895, p. 64) also discussed the perception of wheel-generated motions.

[2] Interest in cycloids was generated from more than just the acknowledged beauty of the curve. The cycloid proved to be the solution for a number of the most pressing mathematical problems of the time. One of these was the attempt to make a more accurate pendulum clock. Although the period of a pendulum, swinging in a circular arch is relatively constant, it is affected by the height from which the bob is dropped. A search was begun to discover the curve that would allow a mass to fall from any position on the curve and reach the bottom in the same length of time. Huygens, in 1673, found this curve to be the inverted cycloid, and clocks were constructed that made use of cycloidal pendulums. Unfortunately Huygen's clocks with cycloidal jaws proved to be no more accurate in operation than those using ordinary simple pendulums. Another puzzle for which the cycloid was found to be the solution was the brachistochrone (Greek for briefest time) problem, proposed by Jean Bernoulli in 1696. It entailed finding the curve that described the fastest path of descent for an object falling in a nonvertical trajectory. The solution is the inverted cycloid, and this principle may be discovered in the tryworks of whaling vessels (Melville, 1851, chap. 46) and also in nature for rapidly eroding hills (Bridge & Beckman, 1977).

cycloidal paths of points on the perimeter of a rolling wheel. Although he did not experiment with simple rolling wheels,[3] he argued from similar demonstrations that one does not perceive the motion of a uniform whole by perceiving individually the movement of its various parts. Duncker (1929/1937) performed the first experiments on the perception of simple wheel-generated motions. His observers saw a wheel roll along a level track in a dark room. When a single light was attached to the perimeter of the wheel, observers saw the light describe a cycloidal path. However, when a second light was placed at the center of the wheel, they perceived a very different phenomenon. Depending in part on the velocity of the wheel, some saw the perimeter point revolving circularly about the center moving in a linear fashion, some saw the perimeter describe loops about the center which again moved with a translatory motion, and others saw the two points of light move like a "tumbling stick" with lights attached to each end.

Most of us, however, became acquainted with studies on the perception of wheel-generated motions only through reading Koffka's (1935) *Principles of Gestalt Psychology.* Koffka drew conclusions about Duncker's (1929/1937) work basing his generalizations on the experiments of Rubin:

> If instead of adding the centre one adds a point on the same concentric circle as the first point, then, to judge from one of Rubin's experiments performed with a somewhat different motion pattern, one can see two such cycloidal motions. If one increases the number of such points, one soon reaches the normal wheel effect, i.e., one sees all points rotating round an invisible centre, and at the same time a translatory motion. (p. 284)

Wallach and Johansson

More recently, Wallach (1965/1976) asserted that if two lights are present on a rolling wheel, diametrically opposite one another on the perimeter, then an observer sees these lights revolving about each other and also moving together with a translatory motion. Wallach did not relate his observation to Koffka's (1935) generalization, but scrutiny of their two accounts reveals an obvious contradiction. Wallach proposed two components to motion perception. First, the revolution of the two points about each other is due to *object-relative displacement.* That is, each point of light moves relative to the other. This object-relative displacement is the rotational component of the observer's motion perception. Second, the translatory motion is determined by *angular displacement.* That is, the pair of lights moves linearly relative to the observer. This angular displacement accounts for the perception of translatory movement.

Johansson (1973), like Duncker (1929/1937) before him, commented on the different perceptions that observers report when viewing (a) a single point of light on the perimeter of a rolling wheel and (b) the case in which another point of light is placed at the wheel's center. In the first case, according to Johansson, the perception is that of a point moving on a cycloidal path, whereas in the latter case, the perception is of the perimeter point following a circular path about the center point which describes a linear motion. Johansson proposed a vector-analytic interpretation for the two components of motion perceived in the latter case. *Linear motion is a common vector description of the motion of both points of light. Extracting this common component of motion from the moving lights, the rotational component is a derived residual.* Johansson specified the essential principle of his perceptual vector analysis as follows: "When, in the motion of a set of proximal elements, equal simultaneous motion vectors can be mathematically abstracted (according to some simple rules), these components are perceptually isolated and perceived as one unitary motion" (p. 205).

Both Wallach (1965/1976) and Johansson (1973) discuss the same two components

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[3] Rubin (1927) rolled a wheel within a ring, the diameter of which was twice that of the wheel. When a single light was present on the perimeter of the wheel that rolled within the ring, an observer sitting in the dark saw the light describe a straight-line pendulum motion, with the greatest speed in the middle. When two lights were present on the wheel he saw two such motions, and it was not until six lights were placed on the wheel that observers saw the motion of a wheel rolling within a ring.

of motion—rotational and translatory—in their interpretations of the perception of wheel-generated movement. However, their accounts differ with respect to the order in which these motion components are extracted from the stimulus event. For Wallach the object-relative displacement is prior; the rotational motion of the lights is extracted first, leaving the linear motion as a residual, which is perceived as angular displacement relative to the observer. For Johansson, on the other hand, the common vector is prior; the linear motion is extracted first, and the rotational motion becomes the residual. If the stimulus event is created by rolling a wheel with two lights on its perimeter, diametrically opposite each other, then both interpretations would predict the same perception, that is, the perception of rotational and translatory movement. However, in the case in which the wheel has a light at its center and one on the perimeter, the two interpretations yield different predictions. Johansson's vector analysis predicts that the linear translatory motion would be extracted as a common vector and the rotational motion of the perimeter light will be left as the residual. Wallach's interpretation predicts that the object-relative displacement of the two lights would be extracted first. *This displacement would describe a rotational motion about a center midway between the two points.* The residual motion, or angular displacement, would describe a prolate cycloid. This is the cycloidal motion of the center between the two points of light. Recall that some of Duncker's (1929/1937) observers described this stimulus event as a "tumbling stick."

Johansson's (1973) vector analysis predicts that lights placed at *any* two points on a rolling wheel will be perceived in terms of two components, linear and rotational motion, the former extracted as a common vector. Wallach's (1965/1976) interpretation places a priority on the extraction of object-relative displacement. Thus, the rotational component of any two lights on a rolling wheel will equal the rotational motion provided by a perceptual vector analysis only in those cases in which the point midway between the two lights is at the center of the wheel. Where these centers do not coincide, angular displacement will be cycloidal. These two views contrast in that if observers extract invariant information in the manner predicted by a perceptual vector analysis, then regardless of where lights might be placed on a rolling wheel, linear translation will be perceived, since it is common to every light. On the other hand, if observers extract first the invariant component of object-relative displacement, then the perception of translational movement will correspond to a metric which describes the invariant parameters of cycloidal angular displacement.

A Metric for Describing Cycloidal Motions

From our experience with these stimuli and from reports of Verbrugge and Shaw (Note 1), we anticipated that Wallach's (1965/1976) account was correct and that observers would judge the movement of some stimulus configurations more wheellike than others. We fashioned a metric that expressed these differences in terms of the parameters of cycloidal angular displacement remaining after the extraction of the invariant rotational component of object-relative displacement. The points of light attached to a wheel form a system of objects which move relative to each other. Consider first a two-light system. One can easily imagine a bar, whose end points are the two lights, spinning and traveling across the screen. Each bar, or system, will have an *apparent* center of moment, which lies midway between the two lights. This is a point around which all movement can be said to occur (Cutting, Proffitt, & Kozlowski, 1978). Our purpose here is to show that this notion works equally well for wheels as it has previously for walkers.

Of course, each system also has an *actual* center of moment, which is the center of the wheel. We propose that it is the relative distance between these two, the apparent and the actual centers of moment, that determines the wheellikeness of these motions given that object-relative displacement is being attended to. More specifically, the nearer these two centers are to one another the more the movement will look like a wheel rolling across a flat surface. On the other hand, the farther apart they are the less it will look like a wheel and the more it will look like a hopping system of objects. We call this measure D_m/r, the *distance* from the *midpoint* of the system of lights to the center of the wheel, expressed as a proportion of the wheel's *radius*. Figure 2 shows how D_m is determined for two- and three-light systems. The length of the dotted lines equals D_m. Figure 3 is a diagram of two cycloids that describe the motions of the centers of a one- and a two-light system. The height of each curve is equal to twice D_m. The horizontal distance traversed in each rotation of the wheel is, of course, equal to $2\pi r$. Our metric thus describes the variable parameters of cycloidal motion for the center of moment of any system of lights on a rolling wheel. Other possible metrics are considered later, as are the location of centers for less and more complex light systems.

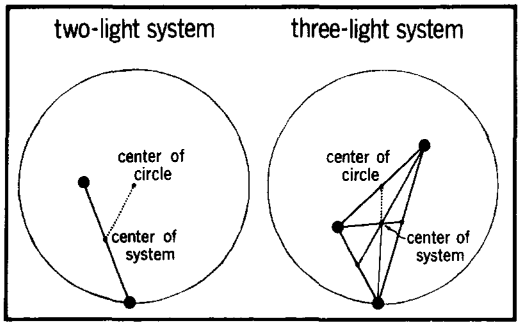

Figure 2. The center of a one-light system is easy to determine, it is the center of the one light. The center of two- and three-light systems is somewhat more difficult to determine. For two lights in rigid relation, the center of the system is the midpoint between the two lights, as shown on the left for Stimulus 8 of Experiment 1. For a three-light system it is the center of the triangle formed by the lights, as shown on the right for Stimulus 8 of Experiment 2. This point is determined by the method of medians. The distance from the center of the system of lights is D_m. When divided by the radius, the normalized distance is D_m/r, and ranges from .00 to 1.00.

Experiment 1: Two-Light Systems

Method

The method of generating point-light stimuli was essentially the same as described in our previous work. It is a modification of Johansson's (1973) second technique, mounting glass-bead retroreflectant tape on moving objects and videorecording them. Bright lights are focused on the object, and the contrast of the image on the television monitor turned to near maximum while the brightness is near minimum. Barclay, Cutting, and Kozlowski (1978) describe this technique in more detail.

Two circular patches of reflectant tape were mounted on the end of a 2-lb. coffee can that had been painted a dark, nonreflectant color. Its diameter was 13 cm. Eleven stimuli were generated by rolling the can across a flat table at a mean of 55 revolutions per minute ($SD = 3.9$). The stimuli differed only in the location of the two patches. One served as a reference light on the perimeter of the can and was not moved. The other was located on the perimeter, halfway between the center and the perimeter, or at the center, as shown in the left panel of Figure 4. Stimuli 1–5 had second lights at the perimeter with 180°, 135°, 90°, 60°, or 30° separating each from the reference lights; Stimuli 6–10 had second lights halfway at 180°, 145°, 90°, 45° or 0° separation; and Stimulus 11 had the second light at the center. (No item other than Stimulus 11 had a center light.) Rolling across the viewing field, stimuli were on monitor for 2.5 revolutions. The diameter of each light was roughly one sixth the diameter of the coffee can. At maximum stimulus height (that for Stimulus 1 vertically aligned) visual angle was between 1° and 2° for all viewers.

A test sequence of 44 trials was produced by rerecording onto a second videotape (see Cutting & Kozlowski, 1977; Kozlowski & Cutting, 1977). This sequence consisted of four consecutive, randomly ordered presentations of all 11 stimuli. Only the last three randomizations were scored; thus, the first served as practice to stabilize use of the judgment scale. The test sequence was played on a

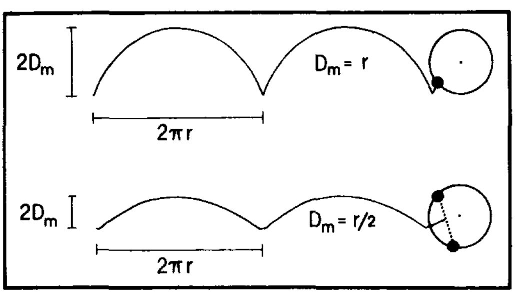

Figure 3. Two cycloidal patterns, one tracing a light at the perimeter and one tracing the center of moment for a two-light system are diagrammed. D_m = the distance from the midpoint of the light system to the center of the wheel; r = the radius of the wheel. Notice that the period of both cycloids is 2π r, yet the depth of vertical excursion and the general shape of the curves differ markedly.

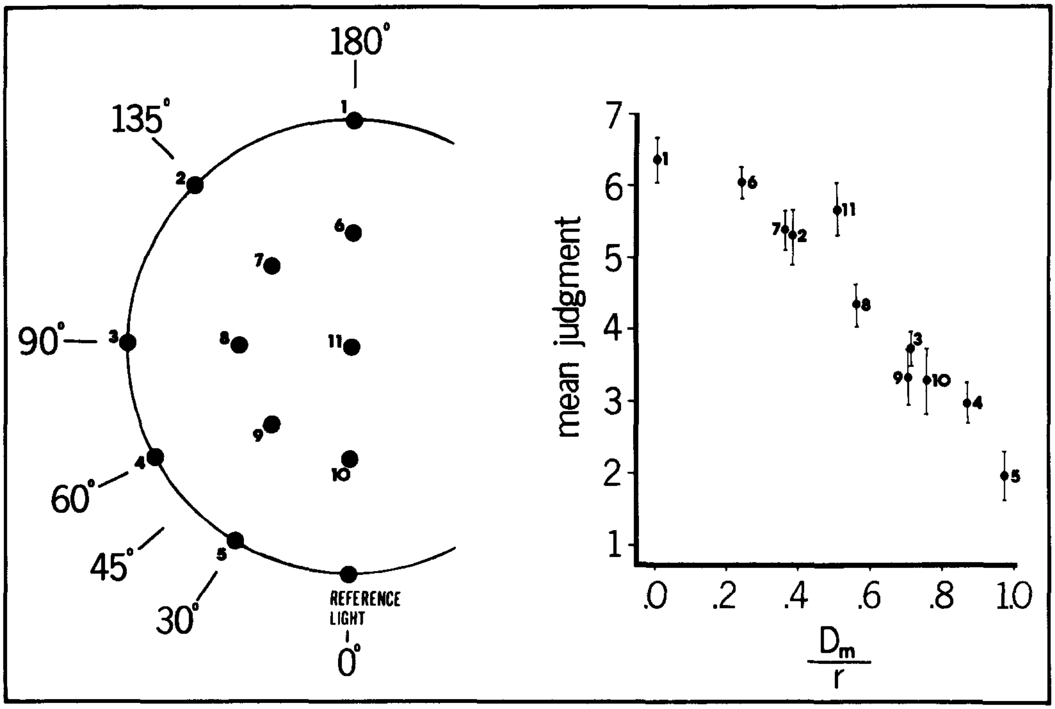

Figure 4. In the left panel, a schematic wheel is shown with the locales of lights for the 11 stimuli in Experiment 1. The reference light is present for all stimuli, and the second light of each stimulus is located at the points designated, with 5 stimuli having the second light on the perimeter, 5 halfway between the center and the perimeter, and 1 at the center. In the right panel are the results of the rating task, with the mean judgment for each stimulus given as a function of the distance of the center of the system of lights from the center of the wheel, D_m/r. Plus and minus one standard error of the mean are given by vertical hatches.

Sony-Matic solid-state videorecorder (Model AV-3650) attached to a 9-in. (22.86 cm) Sony solid-state television (Model PVJ-51RU) serving as the monitor. An audio track announced each trial.

Twelve Wesleyan University summer school students were paid to participate in the study. We told them to use a 7-point unipolar scale to judge "how wheellike the movement appears to be," with 7 indicating the most and 1 the least wheellike movement.

Results and Discussion

The remarkable correspondence between viewers' judgments and the distance measure, D_m/r, is shown in the right panel of Figure 4. The correlation is striking ($r = -.95$, $p < .001$). Moreover, the overall trend is indicative of viewers generally; their coefficient of concordance for the 11 stimuli is very strong ($W = .75$), $\chi^2(10) = 90.0$, $p < .001$ (Siegal, 1956). Thus the distance between apparent and actual centers of moment serves as an excellent index for how movement of the dynamic point-light display is perceived. Following the task, the subjects were asked about how they had used the scale and typically reported that the more the configuration hopped the less wheellike it was rated.

Particular stimulus comparisons. Several intriguing comparisons can be made between certain stimuli. For example, Duncker (1929/1937) and Johansson (1973; Maas & Johansson, 1971) used Stimulus 11—with the second light at the center—as a prototype conveying the most wheellike of rotary motions in a two-light system. In fact, it is *not* the best possible stimulus. Nine of ten viewers (with two ties) rated the movement of Stimulus 1—with lights opposite one another—more strongly wheellike than Stimulus 11 ($z = 2.17$, $p < .03$, two-tailed). Stimulus 11, however, appears to be more strongly wheellike than would be predicted from the linear regression on all points. Its residual, more than one full judgment point, is 2.41 standard deviations greater than the residual mean for all stimuli. Stimulus 11, then, may be a special case, even though not prototypic. One account of these results is that this stimulus can be perceived in two ways. If perceived simply as a two-light system moving about the center, then the D_m/r metric would hold, and a judgment of 4.62 would be predicted. If, on the other hand, the second light was noticed not to have any circular movement component, unlike the second lights of all other stimuli, then it would be perceived as the center, fully specifying rotary movement and suggesting a judgment of 7. Its observed score was 5.68, nearly midway between the two. The notion that systems with a light at the center are better than others is examined more carefully in Experiments 2 and 4.

Perhaps more interesting than possible special cases are the two classes of stimuli, one with second lights on the perimeter and the other with lights halfway out from the center. The rank-order correlation between D_m/r for Stimuli 1–5 and judgments of their rolling motion is perfect ($\rho = 1.00$). It is also perfect for Stimuli 6–10. The value of the distance metric, however, is seen best in the comparison of members of the two sets of stimuli. Other possible measures are simply not as good.

Other metrics for rotary motion? We derived three other possible measures of the stimuli. This seemed necessary because we believe that, in advance, no one metric should be taken as more plausible than another. Moreover, they force careful comparisons between critical pairs of stimuli. Two of these indices employ the relative distance between the two lights. The first of these considers distance alone: Perhaps it would serve as a good predictor of judgments; indeed it does ($r = .79$). As with the previous index, rank-order correlations for Stimuli 1–5 taken as a group and Stimuli 6–10 as a group are both perfect. However, pairwise comparisons across the two sets make this metric suspect: For example, Stimulus 7 has a shorter distance between its two lights than does Stimulus 2, yet it was judged more wheellike. The same is true for Stimulus pairs 8 and 3, 9 and 4, and 10 and 5. Moreover, Stimulus 11 has exactly the same length as Stimulus 4, and yet their scores are quite dissimilar. Our D_m/r metric accounts for all these relations quite nicely.

The second alternative is a combination of the distance metric and D_m/r. If D_m, the distance from the midpoint of the two-light system to the center, is divided by the length of the chord or partial chord formed by that system, then the correlation between this metric and viewer judgments is quite high ($r = -.89$). However, in addition to being less parsimonious than D_m/r, this measure predicts identical judgments for Stimuli 3, 8, and 11, when in fact they differ markedly. Again, our previous index accounts for their relationships well.

Finally, a third metric was considered that might support the relationship between the systems of lights and the wheel's actual center. A perpendicular could be dropped to the wheel's center from the line through the two-point system. Its length, as a function of the length of the radius, might also serve to predict viewer judgments. Crucial to this measure is that it does not matter whether the perpendicular is dropped from the midpoint between the lights (as it would be for Stimuli 1–5), from elsewhere between the lights (for Stimuli 6–8 and Stimulus 11), or from an extension of the line beyond them (as it would be for Stimuli 9 and 10). The correlation between the length of this perpendicular line and viewer judgments, however, is not significant ($r = -.28$). This is primarily due to the fact that it fails to distinguish between Stimuli 1, 6, 10, and 11, which all have zero distance from the center by this method of measurement. Again, our D_m/r index suits these stimuli well.

Thus, in all cases our original metric is superior in its predictions of viewer judgments of wheel-generated motions. It is so good that, barring the special case of Stimulus 11, there is only one reversal—Stimulus

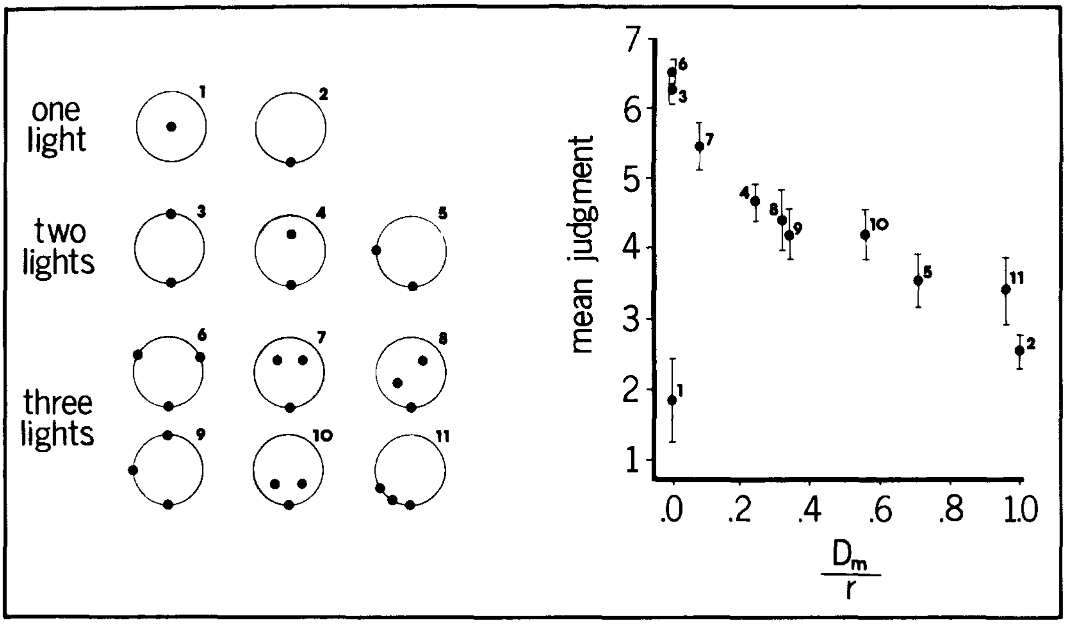

Figure 5. In the left panel are shown the 11 stimuli rated by viewers in Experiment 2, and in the right panel the mean (and standard error) for each stimulus as a function of its systemic distance from the center of the wheel (D_m/r).

3 with Stimulus 9—in an otherwise perfect rank-order correlation. Indeed, given the crude manner with which we fashioned these stimuli, this may not actually be a reversal but may reflect measurement error in making the stimuli.

We next sought to confirm the predictive value of the D_m/r metric with both more and less complex dynamic point-light stimuli.

Experiment 2: One-, Two-, and Three-Light Systems

Method

Procedures were identical to those of the previous experiment. Three old and eight new stimuli were generated, as shown in the left panel of Figure 5. Two were one-light systems, with Stimulus 1 having the light at the center and Stimulus 2 at the perimeter. The distance measures for these two, of course, would simply be their distance from the center as a function of the radius. Thus, they would be .00 and 1.00, respectively. Three stimuli were two-light systems seen previously: Stimulus 3 of this set had perimeter lights 180° apart; Stimulus 4 had lights 180° apart but with one on the perimeter and the other halfway out from the center; Stimulus 5 had perimeter lights 90° apart. (These were Stimuli 1, 6, and 3, respectively, in Experiment 1.) The six other stimuli were three-light systems: Stimulus 6 had lights on the perimeter 120° apart; Stimulus 7 had one perimeter light and two halfway lights at 135° and 225°; Stimulus 8 had two halfway lights at 45° and 225°; Stimulus 9 was identical to Stimulus 3, but with the addition of another light at 90° on the perimeter; Stimulus 10 had two halfway lights at 45° and 315°; and Stimulus 11 had perimeter lights each separated by 30°. The D_m/r for these stimuli was determined from the center of their triangular systems, as measured by the method of medians (see Figure 2).

Twelve Wesleyan University summer school students were paid to view a test sequence of 41 trials. This sequence consisted of four randomizations of Stimuli 2–11, with the 41st trial consisting of the only presentation of Stimulus 1. Viewers used the same 7-point scale as before, and Stimulus 1 was presented last and only once so as not to affect its overall use. Again, the first presentation of the other stimuli served to stabilize judgments and was not counted.

Results and Discussion

Shown in the right panel of Figure 5 is a plot of the judgments of wheellike motion for each stimulus as a function of its D_m/r index. Again, the correlation is quite striking (ignoring the special case of Stimulus 1 for a moment, $r = -.92$, $p < .001$). This value is marginally lower than that for the previous study, but it is perhaps even more remarkable because of the more widely varying stimuli used here. Viewer concordance in judgments was again quite high ($W = .60$), $\chi^2(9) = 64.8$, $p < .001$. Thus, our metric appears to have repeated as a good predictor of viewer performance. With only one reversal—Stimuli 9 and 10—the rank-order correlation for all stimuli is perfect.

Particular stimulus comparisons. Stimulus 1 was judged to be the least wheellike of the 11 stimuli. At first this may appear odd, since its index is zero and since its only light source is directly at the actual center of moment. Its importance here is to demonstrate that a light at the center of the moment is unnecessary to a dynamic point-light display and by itself is utterly insufficient. Coherent movement in a system occurs *about* the center, not *at* the center. This center may move—laterally for the coffee-can stimuli here and both laterally and slightly up and down for the human walkers of Cutting et al. (1978)—but its own movement is not what is crucial to the percept. The viewer perceives the systematic movement about this point in determining the identity of the object-event.

Stimulus 3 and Stimulus 5 yielded judgments similar to those they had in Experiment 1 (where they were Stimuli 1 and 3, respectively). Stimulus 4 of the present study, however, yielded significantly lower judgments than before (as Stimulus 6). This effect is almost certainly due to the fact that in the present study there were three stimuli with lower D_m/r indices, whereas in the previous study there was only one. Given more stimuli that garner more judgments of nearly perfect wheellike motion, it is likely that viewers would adjust their criterion upward as to what type of movements merit high scores. In pneumatic fashion, then, Stimulus 4 of the present study would be assigned lower judgments.

Most satisfying to us is that *three-light systems are not necessarily better stimuli than two-light systems.* Indeed, the multiple correlation of D_m/r and the number of lights in the stimulus showed no contributions in accounting for overall variance by the latter independent variable. For example, there was no significant difference between Stimuli 6 and 3, both with indices of .00. Indeed, *two-light systems can be superior:* Stimulus 3 is judged more wheellike than Stimulus 9, even though the latter stimulus is exactly like the former except that one light is *added* to the perimeter. Thus, this is a clear example where more information yields a less potent stimulus. In Experiment 3, we hold D_m/r constant and make direct comparisons between two- and three-light systems.

Experiment 3: Two- and Three-Light Systems With Equal D_m/r Indices

The results of the previous experiment suggested that the number of lights in the system mattered little in wheellikeness judgments. Since this is contrary to that assumed true by Koffka (1935), we offer this study as a more direct test.

Method

Procedures were the same here as in the previous studies. Five pairs of stimuli, one of each pair with three lights and the other two, were fashioned so that they had equal D_m/r indices. They are shown in the left panel of Figure 6. Stimuli 1, 2, 3, 5, and 9 were used in the previous study, and Stimulus 8 in the first. Stimuli 4, 6, 7, and 10 were matched to them. Twelve viewers from the same population used the scale while viewing a 40-item tape, 4 complete randomizations of the 10 stimuli. Only the last 30 trials were scored.

Results and Discussion

Again, and as shown in the right panel of Figure 6, correspondence between the distance measures and viewer judgments was quite high ($r = -.97$, $p < .001$). Viewer concordance followed suit ($W = .66$), $\chi^2(9) = 71.3$, $p < .001$. Three-light systems garnered marginally higher ratings than two-light systems, $F(1, 11) = 4.65$, $p < .07$, but only the comparisons between Stimuli 1 and 2, and 9 and 10 significantly favored three-light systems, correlated $ts(11) = 3.04$ and 2.64, respectively, $ps < .05$. These effects are minuscule when compared with that for the

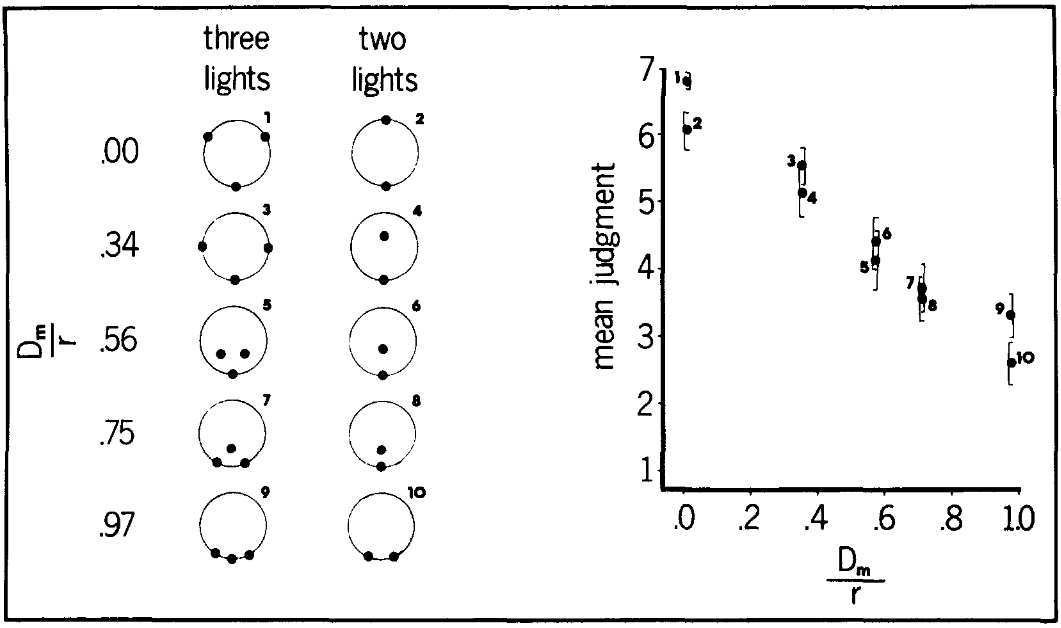

Figure 6. In the left panel are shown the 10 stimuli rated by viewers in Experiment 3, and in the right panel the mean (and standard error) for each stimulus as a function of its systemic distance from the center of the wheel (D_m/r).

D_m/r index, $F(4, 44) = 32.8$, $p < .001$. Thus, perhaps to some small degree Koffka (1935) was correct that, other things being equal, the more lights the better. However, in the present study by multiple correlation, the distance measure accounts for 47 times more variance than does the number of lights in the dynamic system.

Experiment 4: Systems With and Without Center Lights

The results of Experiment 1 suggested that a stimulus with a light at the center might be more wheellike than would be predicted by its distance measure alone. Experiment 2 demonstrated that for a single light this was not true, but we should regard that as a special case. Here, we test the notion directly, holding number of lights and D_m/r constant for pairs of dynamic stimuli with or without central lights.

Method

Three pairs of stimuli were fashioned. All are shown in the left panel of Figure 7. Stimulus 1 was identical to Stimulus 1 of Experiment 1, but with a light at the center. Stimulus 2 was used in Experiments 2 and 3, and Stimulus 3 in Experiment 1. Stimuli 5 and 6 shared common locales for two lights on the perimeter. The 12 viewers of Experiment 3 viewed 18 trials (3 randomizations of the 6 stimuli) immediately following those of the previous study. No trials were discounted.

Results and Discussion

As seen in the right panel of Figure 7, the correlation between D_m/r indices and viewer judgments was high ($r = -.94$, $p < .001$), as was viewer concordance ($W = .47$), $\chi^2(5) = 28.2$, $p < .001$. There was no effect of having a light at the center of the system, $F(1, 11) = .01$, *ns*, although there was the strong effect of distance, $F(2, 22) = 24.6$, $p < .001$. Thus, even in conjunction with other lights, the center position is not special in its contribution to wheellikeness judgments, disconfirming the suspicion raised by the result of Stimulus 11 in Experiment 1.

Experiment 5: Order of Symmetry

The stimuli used in the previous four experiments differed in degree of symmetry.

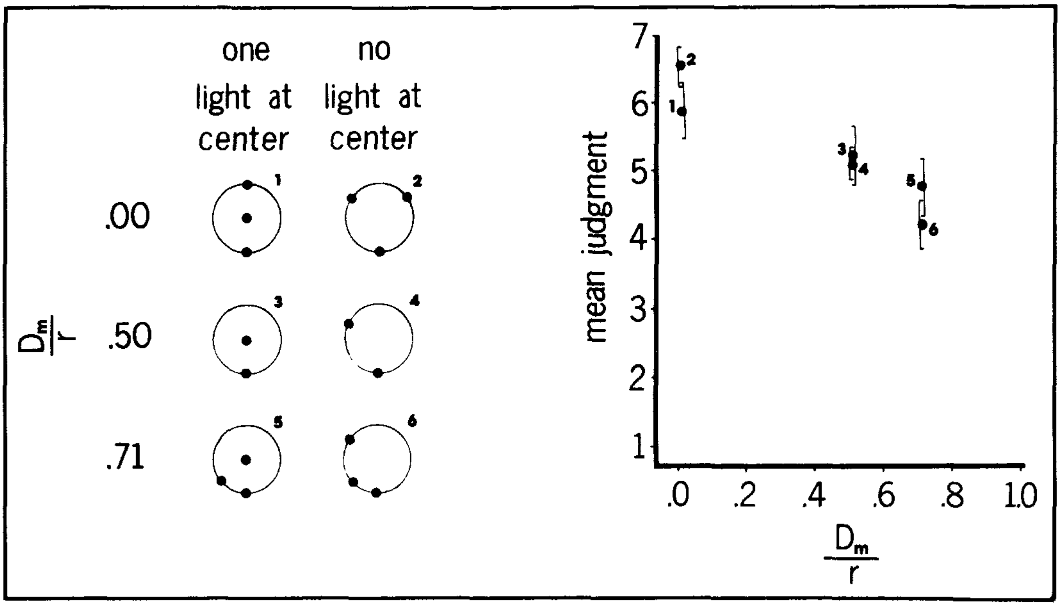

Figure 7. In the left panel are the six stimuli rated by viewers in Experiment 4, and in the right panel the mean (and standard error) for each as a function of its systemic distance from the center of the wheel (D_m/r).

Verbrugge and Shaw (Note 1) suggested that symmetry considerations are important in the perception of simple wheel-generated motions. We offer this last experiment as an examination of symmetry influences.

Rosen (1975, p. 12) defines symmetry as follows: "The general term *symmetry* means invariance under one or more transformations. The more different transformations a system is invariant under, the higher its *degree of symmetry.*" With respect to the transformation of rotation, our stimuli differed in degree of symmetry. Consider the stimulus with three lights on the perimeter, each 120°apart. Rotations by multiples of 120° are symmetry transformations of this stimulus. The degree of symmetry of this stimulus is higher than any other stimulus used. The stimulus with two lights diametrically opposite to each other can be rotated by multiples of 180° and remain invariant; however, all other stimuli used thus far have only multiples of 360° as their symmetry transformations.

The *symmetry group* of a system is defined by the set of all symmetry transformations of that system. Rosen (1975, p. 42) defines the rotational symmetry group as follows: "The set of transformations consisting of the identity transformation and rotations about a given rotation center by $k \times 360°/n$, with n a given integer and $k = 1, 2, \ldots, n-1$, and with consecutive rotation as composition, forms a commutative group called the *cyclic group* of order n and denoted C_n." The *order of symmetry* for each of our stimuli is defined by the integer, n, of its cyclic group. Generally speaking, it is the number of rotations that the stimulus can go through, up to 360°, and remain invariant. The stimulus with three lights 120° apart has an order of symmetry of 3, that with two lights 180° apart has an order of symmetry of 2, and all other stimuli used thus far (except for Stimulus 1 of Experiment 2) have an order of symmetry of 1. This latter case of 1-fold symmetry is the trivial group consisting of only the identity element.

Previously, all of our stimuli with a D_m/r = .00 have had an order of symmetry greater than 1, and other stimuli have had 1-fold symmetry. This experiment introduces stimuli with D_m/r = .00 and orders of symmetry varying from 1 to 4.

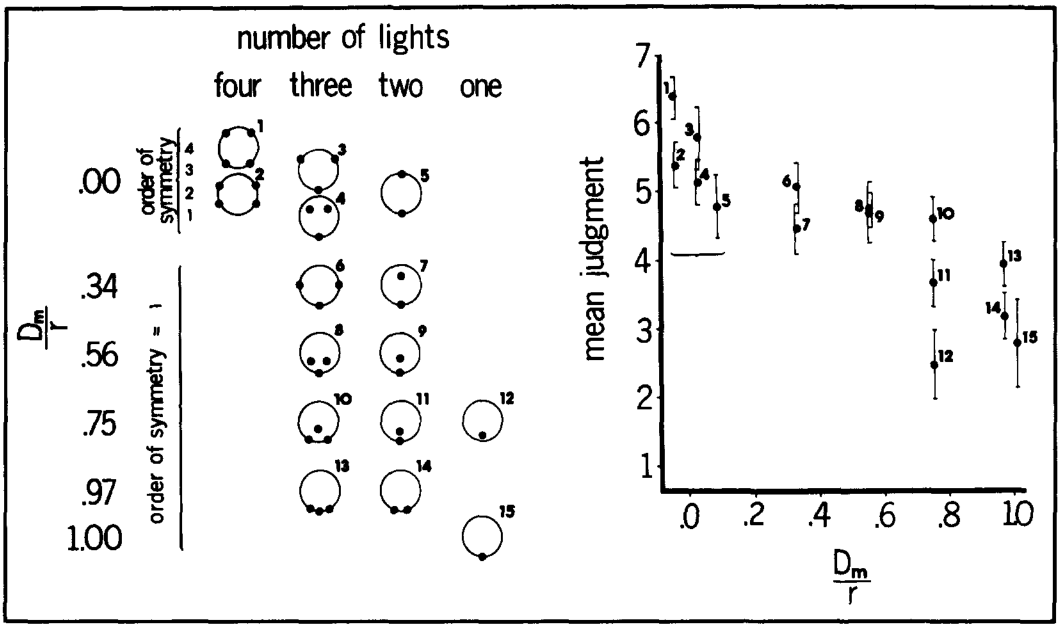

Figure 8. In the left panel are the 15 stimuli rated by viewers in Experiment 5, and in the right panel the mean (and standard error) for each as a function of its systemic distance from the center of the wheel (D_m/r).

Method

Our procedure for generating point-light stimuli was modified slightly. Rather than using reflectant tape, we fashioned our coffee can with small movable lights. The can was rolled across a table and videorecorded as in the previous experiments. Fifteen stimuli were created and are shown in the left panel of Figure 8. Five of these stimuli had D_m/r indices equal to zero. Stimulus 1 had four lights on the perimeter, 90° apart, and thus had an order of symmetry equal to 4. Stimulus 2 had four lights and a 2-fold order of symmetry, Stimulus 3 had three lights and 3-fold symmetry, and Stimulus 5 had two lights and 2-fold symmetry. Stimulus 4 had one light on the perimeter, two lights within, and an order of symmetry of 1. The remaining 10 stimuli (8 of which were used in Experiment 3) all had 1-fold symmetry, varying D_m/r indices, and either one, two, or three lights. Note that an order of symmetry greater than one is possible only when D_m/r is equal to zero.

The test sequence consisted of 60 trials of four randomly ordered presentations of the 15 stimuli. Only the last three randomizations were scored. The presentation differed from those of the previous studies only in that a 23-in. (58.42 cm) Setchell Carson (Model 2100SD) television monitor was used.

Twelve Wesleyan University undergraduate volunteers participated in the study. They were instructed to use the same 7-point scale as was employed previously.

Results and Discussion

The correspondence between viewers' judgments and the distance measure, D_m/r, is shown in the right panel of Figure 8. The correspondence is high ($r = -.82$, $p < .001$). Viewer concordance was high as well ($W = .42$), $\chi^2(14) = 70.6$, $p < .001$. These results thus replicate those of our previous studies.

Order of symmetry cannot, by itself, account for much of the variability in viewer judgments. This is so primarily because order of symmetry can vary only for those stimuli that have a $D_m/r = .00$. If we examine the correspondence between viewer judgments and D_m/r for only those stimuli with 1-fold symmetry, also omitting those stimuli with but one light, we find the correspondence to be fairly high ($r = -.75$, $p < .05$). Comparing the number of lights for each of these nine stimuli and viewer judgments yields a nonsignificant correlation of .48.

For the first five stimuli, with $D_m/r = .00$ and differing orders of symmetry, there appears to be some effect for order of symmetry. The correlation between order of symmetry and viewers' judgments for Stimuli 1–5 is quite high ($r = .87$, $p < .05$). However, the correlation between number of lights and judgments is high as well, although not significant due to the small number of stimuli compared ($r = .69$). Number of lights and order of symmetry are necessarily intercorrelated, since the former defines the upper bound for the latter. The design of this experiment does not allow us to determine the independent contribution of these variables.

Our data suggest that order of symmetry may have an effect on viewers' judgments of wheellike motion; however, this effect is only possible for that subset of stimuli with $D_m/r = .00$. Moreover, the necessary confounding of order of symmetry with number of lights, within this subset of stimuli, makes an accurate estimation of the effect of order of symmetry impossible to determine from our data. That symmetry considerations are not applicable to the majority of our stimuli causes us to subordinate its importance to our analysis in terms of D_m/r.

Conclusion

The results of our five experiments provide strong evidence for the *priority of object-relative displacement* in the extraction of invariant information in event perception. Observers perceive the movement of a small number of lights attached to a rolling wheel by extracting invariant information *in a particular order*. First, the movement of the lights relative to each other is extracted. This object-relative displacement is perceived as a rotation of points of light about a center of moment defined by the system's geometric center. Second, the angular displacement of the system relative to the observer is extracted. This movement is defined by the motion of the center of moment for the system of lights. When this center of moment does not correspond to the center of the wheel, its movement follows a cycloidal path. The invariant parameters of cycloidal angular displacement for any particular system are defined by its D_m/r metric. This metric specifies the distance from the midpoint of the system of lights to the center of the wheel, expressed as a proportion of the length of the wheel's radius. The angular displacement for a system of lights follows a cycloidal path with a height equal to $2D_m$ and a period of 2π r.

We asked our viewers to judge how wheellike the motion of each stimulus appeared and correlated these judgments with the D_m/r metrics. The magnitude of these correlations is striking, and we confess our surprise at the strength of the results. We could find no other metric that accounts for our results as well. Examining the influences of number of lights, order of symmetry, and placement of a light at the wheel's center, we found the effects of the first two to be small and the latter nonexistent.

We are indebted to Johansson's (1973) vector-analytic description of events, but we are in basic disagreement with one of its most essential principles. Johansson's approach provides us with a clear set of terms for describing invariant information extracted in event perception; however, he asserts that common vectors are always extracted first. If this were the case then all of our stimuli would have been perceived as being equally wheellike and having identical rotational and translatory vector components. Our results provide no support for this contention. Rather, we find overwhelming support for Wallach's (1965/1976) position, which emphasizes the priority of the extraction of invariant information specifying object-relative displacement.

We also find some support for Koffka's (1935) and Verbrugge and Shaw's (Note 1) discussions of simple wheel-generated motions. All other things being equal, the greater the number of lights or order of symmetry, the more wheellike will a stimulus appear. These influences probably serve an auxiliary function in specifying centers of moment.

The perception of angular displacement for our wheel-generated motions is defined by the movement of a true abstract entity, the center of moment for the system of lights. Once the invariant information specifying object-relative displacement has been extracted, the motion of the system of objects relative to the observer is specified, not by the motion of any particular object, rather by the movement of the system's center of moment. This center is abstract, since it need not be concretely specified by any light. In fact only one stimulus (Stimulus 1 in Experiment 4) had a light at the system's center of moment, and it was judged somewhat less wheellike than another stimulus (Stimulus 2 of the same experiment) with the same D_m/r metric and number of lights all located on the perimeter.

There are an indefinite number of different mathematical descriptions which could adequately describe the movement of our stimulus lights. The perceptual system selects a particular description by placing a priority on the extraction of object-relative displacement. In so doing, the perceptual system must specify the residual angular displacement in terms of the movement of an abstract entity, the center of moment for the system of objects. Although our D_m/r metric specifies a relationship between two abstract entities—the center of moment for the system of lights and for the wheel—an observer attends to only the former in perceiving the movement of the system. In fact, it is the nonspecification of the wheel's actual center of moment that results in the perception of cycloidal motion.

It would certainly be delightful if future study of the properties of this curve proved as interesting for perceptual psychologists as its study was for mathematicians of the seventeenth century. In subsequent work with wheel-generated motions (Proffitt & Cutting, in press), we use a better mathematical description of geometric centers and continue to examine their perceptual utility.

Reference Note

1. Verbrugge, R. R., & Shaw, R. E. *How we see events: A symmetry analysis of perceptual information for rolling objects.* Paper presented at the 47th Annual Meeting of the Midwestern Psychological Association, Chicago, May 1975.

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Received July 10, 1978 ■